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A RANDOM WALK TREATMENT OF NEUTRON DIFFUSION IN SLABS

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A RANDOM WALK TREATMENT OF NEUTRON DIFFUSION IN SLABS

Technical Report R-482

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by

M. L. Eaton and C. M. Huddleston

ABSTRACT

The concept of a Markov chain has been used to treat the random processes of scatter and absorption which occur when neutrons are incident on a slab of finite thickness, such as a shield or an inside wall of a shelter entranceway. It is assumed that scattering is isotropic in the laboratory system and that the scattering and absorption cross sections do not change during a neutron-scattering history. The encouraging results obtained to date indicate that for thin slabs the analytical random walk method may have important advantages over Monte Carlo calculations (which require lengthy computer runs to obtain acceptably small statistical variances), moments method calculations (which are actually appropriate only for infinite medium cases), and numerical solutions of the neutron transport equation (which are lengthy, tedious, and necessarily approximate).

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The Laboratory invites comment on this report, particularly on the results obtained by those who have applied the information.

CONTENTS

INTRODUCTION

In calculating shielding effectiveness it is often desired to have values for neutron albedo. In particular, the following items are of interest for a neutron incident on a slab of isotropically scattering material:

1. Its total albedo; i.e., its probability of eventually backscattering out of the slab.
2. An analysis of the total albedo into differential albedo as a function of the polar angle of return or backscatter direction.
3. An analysis of the total into the integer numbers of impacts experienced before returning.
4. The probability density of the depth in the slab at which a backscattered neutron makes its last impact.

The classical partial differential transport equation uses time as an independent variable. It appeared that a possible approach to the foregoing set of problems would be to write and solve a psuedo transport equation in which the time scale is replaced by a scale linear in the number of historical neutron impacts. Difficulties in writing such an equation were discussed with Dr. Klaus H. Daniel, Professor, Department of Mathematics, University of Maryland, during his stay at the U. S. Naval Civil Engineering Laboratory in the summer of 1965. Dr. Daniel suggested the possibility of obtaining insight by first obtaining some approximate solutions using increments of slab thickness (instead of differential thickness) and then studying these approximate solutions in the hope of learning how to write the correct equation to go to the limit in the distance variable normal to the slab face.

This suggestion was pursued. There is still no guarantee that the psuedo equation will be written; nevertheless, the method for obtaining approximate solutions by dividing slab thickness into increments seems to hold promise as a usable method in its own right. The description of this method constitutes the purpose of this report.

STATEMENT OF THE PROBLEM

Consider the case of a neutron incident at some polar angle to the surface of a plane slab of material. The slab has a finite thickness and consists of matter which can absorb neutrons or scatter neutrons isotropically in the laboratory system. The probabilities for absorption and scatter at each collision are constant; i.e., the energy of the neutron does not change. The foregoing description is equivalent to the case of a

broad, parallel beam of thermal neutrons incident on a plane slab of a moderator whi has some absorption. It is required to find a probabilistic description of the behavior of the neutrons in the slab.

Mathematically, the problem can be phrased in terms of those quantities which are given and those quantities which are desired.

Given:

1. A plane slab, of thickness T, has known values for Σ_t, Σ_s, and Σ_a, which are, respectively, the total cross section, the scattering cross section, and the absorption cross section. $\Sigma_t = \Sigma_s + \Sigma_a$.

2. Scattering is isotropic in the laboratory system.

3. The slab's dimensions are such that it occupies the space $(-\infty < X < \infty)$, $(-\infty < Y < \infty)$, $(0 \le Z \le T)$. The face at $Z = T$ will be named the forward face, and the initial, near, or backward face is at $Z = 0$.

4. One neutron is directed from the region $(Z < 0)$ toward the origin (on the backward slab face at $Z = 0$) in a course which makes a given angle θ_0 with the $-Z$ axis, and an angle $(\pi/2 - \theta_0)$ with the $-X$ axis.

Find:

1. Total albedo, $\mathcal{A}$ (probability of eventually scattering back to $Z < 0$).

2. Differential albedo, α, as a function of θ, so that

$$\int_0^{2\pi} \int_0^{\pi/2} \alpha(\theta) \sin\theta \, d\theta \, d\phi = \mathcal{A}$$

3. Analysis of total albedo into the number of impacts experienced; i.e.,

$$\mathcal{A} = \sum_{j=1}^{\infty} \mathcal{A}_j$$

4. Analysis of total albedo into depth, Z, $(0 \le Z \le T)$, probability density of the depth at the point of last impact before returning.

5. Total absorption, $\mathcal{A}^\circ$ (probability of eventual absorption of the neutron by the slab).

6. Analysis of $\mathcal{A}^\circ$ into total number of impacts experienced; i.e.,

$$\mathcal{A}^\circ = \sum_{j=1}^{\infty} \mathcal{A}_j^\circ$$

7. Analysis of $\mathcal{a}^{o}$ into depth probability density of point of absorption.
8. Probability, $\mathcal{a}^{t}$, that the neutron will go through the slab eventually.
9. Analysis of $\mathcal{a}^{t}$ into the number of impacts experienced; i.e.,

$$\mathcal{a}^{t} = \sum_{j=0}^{\infty} \mathcal{a}_j^{t}$$

where $\mathcal{a}_o^{t}$ is the probability of uncollided transmission.

10. Analysis of $(\mathcal{a}^{t} - \mathcal{a}_o^{t})$ into depth density of last impact before leaving forward face at $Z = T$.
11. Expected number of neutron impacts, $E(N)$.
12. Expected number of impacts, $E(N_b)$, if neutron is backscattered from the slab.
13. Expected number of impacts, $E(N_o)$, if neutron is absorbed.
14. Expected number of impacts, $E(N_s)$, if neutron is scattered through the slab.
15. Expected number of impacts, $E(N_t)$, if eventually neutron goes through the slab.

For the sake of clarity, a few obvious consequences of the mathematical model are now stated.

Granted that an impact is definitely in the offing, the probability that the neutron will be absorbed at that impact is Σ_a/Σ_t. Granted that it does not become absorbed at that impact, its scattering direction density is uniformly $1/4\pi$ in all directions, so that

$$\int_{\text{sphere}} \frac{1}{4\pi}\, d\Omega = 1$$

or, equivalently,

$$\int_0^{2\pi} \int_0^{\pi} \frac{1}{4\pi} \sin\theta \, d\theta \, d\phi = 1$$

$$\mathcal{a} + \mathcal{a}^{o} + \mathcal{a}^{t} = 1$$

$$\mathcal{a}_o^{t} = e^{-T \Sigma_t \sec\theta_o}$$

$$E(N) = \mathcal{A}\, E(N_b) + \mathcal{A}^{o} E(N_o) + (\mathcal{A}^{t} - \mathcal{A}^{t}_{o})\, E(N_s)$$

$$= \mathcal{A}\, E(N_b) + \mathcal{A}^{o} E(N_o) + \mathcal{A}^{t} E(N_t)$$

$$= \sum_{j=0}^{\infty} j P(j)$$

$$E(N) = \sum_{j=1}^{\infty} j P(j)$$

A PRELIMINARY RESULT

Given a sphere with infinite radius and of material with total cross section Σ_t, given that isotropic scattering of a nonabsorbed neutron has just occurred at a known impact point, and given that an arbitrarily directed W axis with origin at that known impact point has been erected; then, both the scattering angles and the distance to the next impact point are random variables. Also, the W coordinate (perpendicular projection onto the W axis) of the next impact point is a random variable over the range $(-\infty < W < \infty)$. Let the units for reckoning distance be the same in the W system as in the (X, Y, Z) system.

Assuming the foregoing, consider the following problem: Find the density function, h(W), for the W coordinate of the next impact. For motivation, it is mentioned now that in sections to follow the W axis will be oriented parallel to the Z axis for the slab problem.

Because this whole report depends upon the validity of this function, h(W) will be sought by two different methods: first, directly; and second, indirectly, by computing first the distribution function H(W), and then differentiating it with respect to W to obtain h(W).

Before starting either solution, some preliminary comments seem appropriate. By a distribution function, H(W), is meant a function bounded below by zero and above by unity such that

$$H(W) = \int_{-\infty}^{W} h(\xi)\, d\xi$$

and $dH(W)/dW = h(W)$, everywhere that H(W) is differentiable, where h(W) is the density function of the random variable W.

Because of symmetry about the point $W = 0$, $H(0) = 1/2$, and $h(W) = h(-W)$. Finally, the following general properties of all distribution functions are noted:

$$H(-\infty) = 0$$

$$H(\infty) = 1$$

H(W) is monotonically nondecreasing with increasing W.

Direct Derivation of h(W)

Consider any ray from the origin $W = 0$, the immediately previous impact point, into the right half infinite sphere $W > 0$, making an angle $\beta < \pi/2$ with the positive W axis. For a neutron traveling this ray, the W coordinate of the next impact is a random variable with density function $g(W) = \Sigma_t \sec\beta \, e^{-\Sigma_t W \sec\beta}$, $W > 0$. Since this is true for <u>all</u> such rays in the right half sphere, their uniformly $(1/4\pi)$ weighted "sum" is

$$h(W) = \int_0^{2\pi} \int_0^{\pi/2} \frac{\Sigma_t}{4\pi} \sec\beta \, e^{-\Sigma_t W \sec\beta} \sin\beta \, d\beta \, d\eta$$

where η represents angles in the plane perpendicular to the W axis through the point $W = 0$. This recognizes the fact that scatters from this impact point into the left half sphere or the right half sphere are equally likely.

Thence, by integrating over η,

$$h(W) = \frac{\Sigma_t}{2} \int_0^{\pi/2} \tan\beta \, e^{-\Sigma_t W \sec\beta} \, d\beta$$

To facilitate this remaining integration, the following change of variable is made:

$$V = \Sigma_t W \sec\beta$$

$$dV = \Sigma_t W \tan\beta \sec\beta \, d\beta$$

$$\frac{dV}{V} = \tan\beta \, d\beta$$

Substitution gives

$$h(W) = \frac{\Sigma_t}{2} \int_{\Sigma_t W}^{\infty} \frac{e^{-V}}{V} dV$$

$$h(W) = -\frac{\Sigma_t}{2} \, Ei(-\Sigma_t W) \qquad W > 0$$

By symmetry,

$$h(W) = -\frac{\Sigma_t}{2} \, Ei(\Sigma_t W) \qquad W < 0$$

Values for the exponential integral Ei(·) are tabulated.[1,2]

Indirect Derivation of h(W)

Noting that

$$H(0) = \frac{1}{2}$$

and that for any neutron leaving the origin W = 0 and entering the right half sphere, the probability of its impacting again, before its projection on the W axis becomes as large as any particular positive W value, is $1 - e^{-\Sigma_t W \sec\beta}$,

$$H(W) = \frac{1}{2} + \frac{1}{2} \int_0^{\pi/2} \left(1 - e^{-\Sigma_t W \sec\beta}\right) \sin\beta \, d\beta$$

$$= 1 - \frac{1}{2} \int_0^{\pi/2} e^{-\Sigma_t W \sec\beta} \sin\beta \, d\beta \qquad W > 0$$

$$= 1 - \frac{W\Sigma_t}{2} \int_{\Sigma_t W}^{\infty} \frac{e^{-V}}{V^2} \, dV$$

$$= 1 - \frac{e^{-\Sigma_t W}}{2} - \frac{\Sigma_t W}{2} \, Ei(-\Sigma_t W) \qquad W > 0$$

Because of symmetry, $H(\infty) - H(W) = H(-W) - H(-\infty)$, and thus,

$$H(W) = \frac{e^{\Sigma_t W}}{2} - \frac{\Sigma_t W}{2} Ei(\Sigma_t W) \qquad W < 0$$

$$h(W) = \frac{dH(W)}{dW}$$

$$h(W) = \frac{\Sigma_t}{2} e^{-\Sigma_t W} - \frac{\Sigma_t}{2} Ei(-\Sigma_t W) - \frac{\Sigma_t}{2} e^{-\Sigma_t W} \qquad W > 0$$

$$h(W) = -\frac{\Sigma_t}{2} Ei(-\Sigma_t W) \qquad W > 0$$

$$h(W) = -\frac{\Sigma_t}{2} Ei(\Sigma_t W) \qquad W < 0$$

INTRODUCTION TO MARKOV CHAIN CONCEPTS

Consider some phenomenon in which a "particle" at any one instant may be in any one of a known number (few, several, many, or a countably infinite number) of different possible states (positions or conditions). Given that the particle can change its state sequentially in a chain of steps, and that for each existing state, the probability of its being in any other state immediately after the next step is known for all states; then the problem of computing for a particle in one known given state the probability of its being in some particular given state exactly n steps later is a Markov chain problem.

The following example may clarify the foregoing abstraction. Bill has four coins and Joe has three coins, and they agree to gamble these, one at a time (say by matching or flipping) where for each person, the probability of winning or losing the coin each time (each step) is one-half. From Joe's point of view there are eight possible states: broke, has one coin, ..., has six coins, has seven coins. The chain of steps is the successive encounters or matchings with the attendant change in wealth. It is mathematically convenient to indicate the different possible states as:

(1,0,0,0,0,0,0,0)	means Joe is broke (Bill has seven coins)
(0,1,0,0,0,0,0,0)	means Joe has one coin
(0,0,1,0,0,0,0,0)	means Joe has two coins
⋮	
(0,0,0,0,0,0,0,1)	means Joe has seven coins (Bill is broke)

Here the "particle" is the 1 in the vector, and the state of Joe's wealth at the close of any event is indicated by the position of the particle. The game starts with the state vector being $V_o = (0,0,0,1,0,0,0,0)$, i.e., with Joe having three coins, and will continue through a sequence of vectors indefinitely, until either Joe goes broke $(1,0,0,0,0,0,0,0)$ or Bill goes broke $(0,0,0,0,0,0,0,1)$.

If the game is just about to start, we cannot know now the state of Joe's wealth, say, three encounters later. However, for each of the eight states, it is possible now to compute the probability that he will be in that particular state at the close of the third encounter. To make this and other computations, first a transition matrix M is displayed.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

To explain the construction of the matrix, it is noted that there are eight rows and eight columns; and thus, there are 64 entries within the matrix. If $i = 0, 1, \ldots, 7$, and $j = 0, 1, \ldots, 7$, and p_{ij} are the entries, then any p_{ij} appearing in the i^{th} row and j^{th} column is simply the answer to the following question: "given that the particle is in i^{th} state, what is the probability that <u>at</u> (<u>immediately</u> after) the next encounter (step) the particle will be in state j?" For example, if at some instant Joe has six coins, then as an immediate result of the next matching he will have either five or seven coins, each with probability 1/2; and zero, one, two, three, four, or six are impossible (probability is zero). These facts are indicated in the next to last row of M by the ordered entries $0\ 0\ 0\ 0\ 0\ \frac{1}{2}\ 0\ \frac{1}{2}$.

Now to show the use of M, we perform the multiplication $V_o M$ to obtain V_1, the state vector, just after the first matching.

$$V_1 = V_o M$$

$$= (0\ 0\ \tfrac{1}{2}\ 0\ \tfrac{1}{2}\ 0\ 0\ 0)$$

This result merely shows that Jce is equally likely to have two or four coins after the first step, and it is impossible to have at that time any other sum.

Now, performing the multiplication $V_1 M$,

$$V_2 = V_1 M$$

$$V_2 = (0 \ \tfrac{1}{4} \ 0 \ \tfrac{1}{2} \ 0 \ \tfrac{1}{4} \ 0 \ 0)$$

which simply means that at the end of the second step Joe's probabilities are as follows:

P (broke) = 0

P (single coin) = $\frac{1}{4}$

P (two coins) = 0

P (three coins) = $\frac{1}{2}$

P (four coins) = 0

P (five coins) = $\frac{1}{4}$

P (six coins) = 0

P (seven coins) = 0

Continuing,

$$V_3 = V_2 M$$

$$V_3 = (\tfrac{1}{8} \ 0 \ \tfrac{3}{8} \ 0 \ \tfrac{3}{8} \ 0 \ \tfrac{1}{8} \ 0)$$

It is noted that at each step the sum of the entries in the state vector is unity, as required by the fact that the "particle" must at all times be somewhere.

It is possible to continue indefinitely and observe that as the number of steps increases the state vector will approach the inevitable result:

$$V_\infty = (\tfrac{4}{7} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \tfrac{3}{7})$$

This simply means that if Joe and Bill agree to play until one is broke the probability that Joe goes broke eventually is $\frac{4}{7}$.

A MARKOVIAN APPROXIMATION TO THIN-SLAB ALBEDO

Returning now to the problem of this report, it is noted first that the distance from the slab face to a neutron impact point for some future impact of interest is not exactly amenable to the Markov approach described above, because this distance is a continuous variable in contradistinction to a discrete variable. Thus, the number of possible states is uncountably infinite. Nevertheless, it is proposed to obtain approximate results by dividing (conceptually) the slab thickness into an integer number $(\frac{T}{h})$ of equally thin sub-slabs (cells), each of thickness h. This gives rise to $\frac{T}{h} + 2$ Markov states numbered i or $j = 0, 1, \ldots, \frac{T}{h}, \frac{T}{h} + 1$.

At any step (serially numbered neutron impact), the existence of the neutron in state (or cell) zero means by convention that it has already returned to region $Z < 0$ and, thus, will never actually experience that numbered impact. At any numbered impact (step), state five means that the neutron has experienced that impact at some point such that $4h < Z < 5h \leq T$. At any step the neutron in state $(\frac{T}{h} + 1)$ means by convention that it has already passed through the slab of thickness T and will never experience that or later impacts.

The reason this approach will not be exact is that when computing a typical element m_{ij} of the $(\frac{T}{h} + 2)$ by $(\frac{T}{h} + 2)$ transition matrix M, the assumption is made that if the last impact occurred in the i^{th} cell, it occurred at a midpoint of this cell. This tends to err on the side of stability, causing the diagonal elements m_{ii} to be slightly too large. It is expected, however, that as $h \to 0$, i.e., as the number of cells is increased, the computed albedo, as well as other desired results, will converge rapidly so as to produce results with sufficient accuracy for practical purposes.

If by convention the $(\frac{T}{h} + 2)$ elements of state vector V_1 are the probabilities that the first impact will occur in the corresponding cell, then it will be possible to compute V_1 exactly (within round-off error).

The next section of this report consists of a specific numerical example. There are two reasons for presenting such a section. First, the authors find that it is easier to explain the concepts by an example than by an abstraction. Second, it is believed that it will be easier to follow.

A NUMERICAL EXAMPLE

In order to achieve a better understanding of the physical problem, let us consider a neutron normally incident on a slab whose thickness is one half of a mean free path. Let us subdivide the slab conceptually into five layers each of thickness h. Assume the absorption cross section is one fourth as large as the scattering cross section. Then we have

$$T = \frac{1}{2\Sigma_t}$$

$$\frac{\Sigma_a}{\Sigma_t} = 0.2$$

$$\theta_o = 0$$

$$h = \frac{1}{10\,\Sigma_t}$$

$$i = 0, 1, \ldots, 5, 6$$

The "cells" are the various slab regions. For example, the zeroth cell, or region, is the void which the incident neutron traverses before it reaches the slab. The first cell is the first layer of the slab which the neutron encounters as it enters the slab. The various cells, then, are defined as follows:

$i = 0$	$Z < 0$
$i = 1$	$0 < Z < 0.1$
$i = 2$	$0.1 < Z < 0.2$
$i = 3$	$0.2 < Z < 0.3$
$i = 4$	$0.3 < Z < 0.4$
$i = 5$ $(\frac{T}{h} = 5)$	$0.4 < Z < 0.5$
$i = 6$	$0.5 < Z$

One unit of distance (for example along the Z axis) is equal to $1/\Sigma_t$.

It will be noted that a point on a cell boundary is left ambiguous, and the authors without apology take the dodge that an impact will occur there with probability zero. It is desired to approximate the fifteen items listed in this report under the heading "Statement of the Problem."

The V_1 state vector, showing the position by cell number for the first impact, will now be computed. The probability that this will occur in cell zero is zero, because by assumption the space $Z < 0$ is vacuum except for the neutron moving along the negative Z axis.

$$P\,(1^{st}\text{ impact in cell 1}) = \int_0^{\frac{1}{10\Sigma_t}} \sec\theta_o\,\Sigma_t\, e^{-\sec\theta_o\,\Sigma_t\, Z}\, dZ$$

$$= 0.0951626$$

$$\vdots$$

$$P\,(1^{st}\text{ impact in cell 5}) = \int_{\frac{4}{10\Sigma_t}}^{\frac{5}{10\Sigma_t}} \sec\theta_o\,\Sigma_t\, e^{-\sec\theta_o\,\Sigma_t\, Z}\, dZ$$

$$= 0.0637894$$

$$P\,(\text{cell 6 entered with no impact}) = \int_{\frac{5}{10\Sigma_t}}^{\infty} \sec\theta_o\,\Sigma_t\, e^{-\sec\theta_o\,\Sigma_t\, Z}\, dZ$$

$$= 0.6065307$$

Thus, the transpose of state vector V_1 is

$$V_1' = \begin{bmatrix} 0 \\ 0.0951626 \\ 0.0861067 \\ 0.0779125 \\ 0.0704982 \\ 0.0637894 \\ 0.6065307 \end{bmatrix}$$

This simply means that, a priori, the position of the first impact of a neutron with a nucleus of the slab material is unknown, but nonetheless the probability that this will occur, for example in the fourth cell, is 0.0704982 or, equivalently, about 7.05% of a large number of neutrons will experience their first impact somewhere within cell number four.

As a prelude to the construction of the transition matrix M, it will be recalled that for an arbitrary directional one-dimensional W coordinate system with origin at a last point of impact from which an unabsorbed neutron in leaving isotropically (in three space), the distribution function of the W coordinate of the next impact point is

$$H(W) = \begin{cases} \dfrac{e^{\Sigma_t W}}{2} - \dfrac{\Sigma_t W}{2} \, Ei\,(\Sigma_t W) & W < 0 \\ \dfrac{1}{2} & W = 0 \\ 1 - \dfrac{e^{-\Sigma_t W}}{2} - \dfrac{\Sigma_t W}{2} \, Ei\,(-\Sigma_t W) & W > 0 \end{cases}$$

Let us now measure W coordinates, h, and T in distance units equal to $(1/\Sigma_t)$ and use the above expression, a table of the exponential function, and a table of the Ei(·) function to compute Table 1 for H(W) versus W.

Table 1. Tabulation of Values of Distribution Function H(W) Versus W

W	H(W)	W	H(W)	W	H(W)	W	H(W)
-0.51	0.1605531	-0.37	0.2056105	-0.23	0.2696302	-0.09	0.3706221
-0.50	0.1633219	-0.36	0.2094350	-0.22	0.2752676	-0.08	0.3804805
-0.49	0.1661514	-0.35	0.2133564	-0.21	0.2810874	-0.07	0.3909176
-0.48	0.1690435	-0.34	0.2173784	-0.20	0.2871003	-0.06	0.4020231
-0.47	0.1719999	-0.33	0.2215052	-0.19	0.2933180	-0.05	0.4139172
-0.46	0.1750229	-0.32	0.2257046	-0.18	0.2997535	-0.04	0.4267694
-0.45	0.1781145	-0.31	0.2300901	-0.17	0.3064211	-0.03	0.4408360
-0.44	0.1812770	-0.30	0.2345576	-0.16	0.3133370	-0.02	0.4565523
-0.43	0.1845127	-0.29	0.2391486	-0.15	0.3205194	-0.01	0.4748353
-0.42	0.1878240	-0.28	0.2438687	-0.14	0.3279889	0.00	0.5000000
-0.41	0.1912135	-0.27	0.2487238	-0.13	0.3357693	+0.01	0.5251647
-0.40	0.1946840	-0.26	0.2537203	-0.12	0.3438877	+0.02	0.5434477
-0.39	0.1982383	-0.25	0.2588651	-0.11	0.3523762	+0.03	0.5591640
-0.38	0.2018794	-0.24	0.2641657	-0.10	0.3612725	+0.04	0.5732306

At each impact the neutron is either absorbed permanently then and there (with probability $\Sigma_a/\Sigma_t = 0.2$), or it moves isotropically in three-space from there (with probability $\Sigma_s/\Sigma_t = 0.8$). To assist in computing the m_{ij} elements of M, it is convenient to orient a W axis parallel to the Z axis with W increasing with increasing Z and with the origin of the W axis at the center* of the i^{th} cell. Then, Table 1 is used to compute

$$m_{ij} = \frac{\Sigma_s}{\Sigma_t}\left\{H\left[\left(j - i + \frac{1}{2}\right)h\right] - H\left[\left(j - i - \frac{1}{2}\right)h\right]\right\} \qquad \begin{array}{l} i = 1, \ldots, \frac{T}{h} \\ j = 1, \ldots, \frac{T}{h} \end{array}$$

$$m_{io} = \frac{\Sigma_s}{\Sigma_t} H\left[\left(i - \frac{1}{2}\right)h\right] \qquad i = 1, \ldots, \frac{T}{h}$$

$$m_{i,\frac{T}{h}+1} = \frac{\Sigma_s}{\Sigma_t} H\left[\left(i - \frac{T}{h} - \frac{1}{2}\right)h\right] \qquad i = 1, \ldots, \frac{T}{h}$$

$$m_{o,j} = \begin{cases} 0 & j = 1, \ldots, \frac{T}{h} + 1 \\ 1 & j = 0 \end{cases}$$

$$m_{\frac{T}{h}+1,\,j} = \begin{cases} 0 & j = 0, 1, \ldots, \frac{T}{h} \\ 1 & j = \frac{T}{h} + 1 \end{cases}$$

The foregoing seemingly confusing formulae may be clarified by referring to Figure 1, which is a geometric display of the cells for this numerical example. The orientation of the illustration is across the slab perpendicular to the faces and shows the cell locations (Z coordinates of the edges and center of the five interior cells). The thickness of the slab is $0.5/\Sigma_t$.

* All the inaccuracy of this method stems from this approximation; i.e., the mean effective center of impact for the cell is not necessarily at the center.

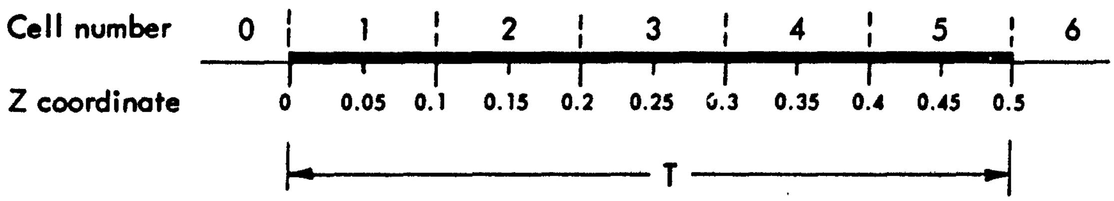

Figure 1. Cell number.

Now, for example, to compute m_{41}, referring to Figure 1, it is noted that for the neutron to move from cell 4 with center at Z = 0.35 to cell 1, it is necessary to move to the left (decreasing Z) a distance someplace between 0.25 and 0.35 units before the next impact. Now, referring to Table 1, H(−0.25) - H(−0.35) = 0.2588651 - 0.2133564 = 0.0455087. But for any impact in cell 4, the probability of immediate neutron absorption there is 0.2. Thus,

$$m_{41} = 0.8(0.0455087) = 0.0364070$$

The complete transition matrix M is displayed in Table 2.

Table 2. Transition Matrix M $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

$$\begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.3311338 & 0.1377324 & 0.0747183 & 0.0493234 & 0.0364070 & 0.0281935 & 0.1424916 \\
0.2564155 & 0.0747183 & 0.1377324 & 0.0747183 & 0.0493234 & 0.0364070 & 0.1706851 \\
0.2070921 & 0.0493234 & 0.0747183 & 0.1377324 & 0.0747183 & 0.0493234 & 0.2070921 \\
0.1706851 & 0.0364070 & 0.0493234 & 0.0747183 & 0.1377324 & 0.0747183 & 0.2564155 \\
0.1424916 & 0.0281935 & 0.0364070 & 0.0493234 & 0.0747183 & 0.1377324 & 0.3311338 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0
\end{bmatrix}$$

Using V_1 and M the following sequential computations are performed:

$$V_2 = V_1 M$$

$$V_3 = V_2 M$$

$$V_4 = V_3 M$$

$$\vdots$$

This process should converge reasonably rapidly to V_∞, a limiting vector in which the zeroth component represents total albedo, the sixth component represents probability of transmission through the slab, and the other components are zero. For any state vector V_k, one minus the sum of the components is the probability of neutron absorption before the k^{th} impact. Table 3 displays the results of the first few $V_{k+1} = V_k M$ computations.

Table 3. Successive State Vectors $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

V_1	0.0	0.0951626	0.0861067	0.0779125	0.0704982	0.0637894	0.6065307
V_2	0.0908481	0.0277487	0.0305911	0.0302724	0.0280093	0.0237141	0.6901224
V_3	0.1223097	0.0092891	0.0107940	0.0110863	0.0104107	0.0087482	0.7206015
V_4	0.1334728	0.0032584	0.0038411	0.0040010	0.0037865	0.0031845	0.7316297
V_5	0.1374653	0.0011608	0.0013742	0.0014388	0.0013665	0.0011506	0.7356036
V_6	0.1388972						0.7370328
.							
.							
V_∞ (est.)	0.13979	0.0	0.0	0.0	0.0	0.0	0.73792

Estimated probability of absorption = 1 - 0.13979 - 0.73792 = 0.12229.

In the process of computing the zeroth component of any V_{k+1} vector, using the row by column rule, it is possible to note in detail from which cells the increase in total albedo springing from the k^{th} impact came. This detail is recorded in Table 4. For example, the probability that the neutron will enter the zeroth cell immediately after a final fourth impact in cell number 2 is simply

$$0.0038411 \times 0.2564155 = 0.00098492$$

where 0.0038411 is the number 2 cell component of V_4 (Table 3), and 0.2564155 is m_{20} read from Table 2. The result, 0.0009849, is recorded in Table 4. The last line of Table 4 contains Q values, which are the analysis of total albedo into the cell of final impact.

In a similar manner, Table 5 is a record of entrances into cell number 6, maintained by impact number and cell source.

Total albedo (probability of entering cell zero), or proportion of a very large number of incident neutrons which backscatter, consists of those which have experienced exactly one impact, exactly two impacts, exactly three impacts, etc. From the left column of Table 3, by taking differences, it is possible to construct Table 6, which is a breakdown of neutrons entering cell zero into the number of impacts before returning to the region $Z < 0$. Of course, it is computationally expedient during the matrix multiplication leading to Table 3 to record entries in Table 6. (This avoids the need for actual subtraction.)

Table 4. Probability of Neutron's Entering Zeroth Cell, Having Had i Impacts With i^{th} Impact Occurring in Cell j $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

i	j = 1	2	3	4	5	Total
1	0.0315115	0.0220791	0.0161351	0.0120330	0.0090894	
2	0.0091885	0.0078440	0.0062692	0.0049537	0.0033791	
3	0.0030759	0.0027678	0.0022959	0.0017769	0.0012465	
4	0.0010790	0.0009849	0.0008286	0.0006463	0.0004538	
5	0.0003844	0.0003524	0.0002980	0.0002332	0.0001639	
$\sum_{i=1}^{5} p_{ij}$						
	0.0452393	0.0340282	0.0258267	0.019643	0.0143328	0.1390503
Est $\sum_{i=6}^{\infty} p_{ij}$						
	0.00020	0.00018	0.00015	0.00012	0.00009	
Est $\sum_{i=1}^{\infty} p_{ij} = Q_j$						
	0.04544	0.03421	0.02598	0.01976	0.01442	0.13981

Table 5. Probability of Neutron's Entering 6th Cell, Having Had i Impacts With i^{th} Impact Occurring in Cell j $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

i	j = 0	1	2	3	4	5	Total
0	0.6065307	0.0	0.0	0.0	0.0	0.0	
1	0.0	0.0135599	0.0146971	0.0161351	0.0180768	0.0211228	
2	0.0	0.0039540	0.0052215	0.0062692	0.0071820	0.0078526	
3	0.0	0.0013236	0.0018424	0.0022959	0.0026695	0.0028968	
4	0.0	0.0004643	0.0006556	0.0008286	0.0009709	0.0010545	
5	0.0	0.0001654	0.0002346	0.0002980	0.0003504	0.0003810	
$\sum_{i=1}^{5} P_{ij}$							
	0.6065307	0.0194671	0.0226511	0.0258267	0.0292496	0.0333077	0.7370329
Est $\sum_{i=6}^{\infty} P_{ij}$							
		0.00013	0.00015	0.00018	0.00020	0.00023	0.00089
Est $\sum_{i=1}^{\infty} P_{ij}$							
	0.6065307	0.01960	0.02280	0.02601	0.02945	0.03354	0.73793

Table 6. Number of Impacts Analysis of Albedo (Zeroth Cell Entrants)

$$\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$$

Number of Impacts	Probability
1	0.0908481
2	0.0314616
3	0.0111631
4	0.0039925
5	0.0014319
Total of 5	0.1388972
6 and more (est.)	0.00089
Estimated total albedo	0.13979

In a similar manner, the estimated number of historic impacts for neutrons entering cell 6 (passing through the slab never to return) are listed in Table 7.

Table 7. Number of Impacts Analysis of Neutrons Transmitted Through Slab

$$\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$$

Number of Impacts	Probability
0	0.6065307
1	0.0835917
2	0.0304791
3	0.0110282
4	0.0039739
5	0.0014293
Total 0 to 5 inclusive	0.7370328
6 and more (est.)	0.00089
Estimated total through slab	0.73792

In order to break down the 0.12229 absorbed (beneath Table 3) into the impact number (i.e., first, second, third, etc.) at which absorption takes place, Table 8 is constructed by first summing the components of the successive state vectors (Table 3) and then subtracting to find the amounts of absorption at successive impacts.

Table 8. Absorption of Neutrons by Cell and Impact Number

$$\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$$

Impact Number (i)	$\sum_{j=0}^{6} v_{ij}$	Absorbed	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5
1	1.0000000	0.0786939	0.0190325	0.0172213	0.0155825	0 0140996	0.0127579
2	0.9213061	0.0280666	0.0055497	0.0061182	0.0060545	0.0056018	0.0047428
3	0.8932395	0.0100657	0.0018578	0.0021588	0.0022173	0.0020821	0.0017496
4	0.8831739	0.0036143	0.0006517	0.0007682	0.0008002	0.0007573	0.0006369
5	0.8795596	0.0012981	0.0002322	0.0002748	0.0002878	0.0002733	0.0002301
Est 6, 7, 8, ...		0.00055	0.00011	0.00011	0.00011	0.00011	0.00011
Est ∞	0.87771						
Total		0.12229	0.02743	0.02665	0.02505	0.02292	0.02023

The foregoing computations have been concerned in the main with proportion absorbed, proportion passing through, and total albedo. It is now time to come to grips with the problem of analyzing the total albedo with reference to the directional (θ) density, $g(\theta)$, of backscatter and, of even more interest to the physicist, the differential albedo $\alpha(\theta, \phi)$ for $0 < \theta < \pi/2$. It is essential to note that these differ from each other.

First, define $\mathcal{A}$, the total albedo, to be the probability that eventually the neutron backscatters to $Z < 0$. This is, of course, a function of Σ_a, Σ_s, T, and θ_o. Then α, the differential albedo per steradian is defined so that

$$\int_{BHS} \alpha \, d\Omega = \mathcal{A}$$

where BHS is the back hemisphere in the region $Z < 0$, with base in the plane $Z = 0$, and $d\Omega$ is the differential solid angle so that

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

The defining relation can be rewritten as

$$\int_0^{2\pi}\int_0^{\pi/2} \alpha \sin\theta \, d\theta \, d\phi = \mathcal{Q}$$

In general, α is a function of both θ and ϕ. For the isotropic case of this report, α is independent of ϕ; nevertheless, it is well to remember that $\alpha/\mathcal{Q}$ is a two-dimensional directional density of the backscattered neutron. After the first integration is performed,

$$2\pi \int_0^{\pi/2} \alpha \sin\theta \, d\theta = \mathcal{Q}$$

or

$$\int_0^{\pi/2} \mathcal{Q} \, g(\theta) \, d\theta = \mathcal{Q}$$

where

$$g(\theta) = \frac{2}{\mathcal{Q}} \pi \alpha \sin\theta$$

a density which displays the relative frequency of return at all the values of θ, $0 < \theta < \pi/2$.

For a neutron that is backscattered, having had its last impact at a known depth Z, the angles (θ, ϕ) of return are, of course, random variables with some joint density, say $f(\phi, \theta)$, where

$$\int_{BHS} f(\phi, \theta) \, d\Omega = 1$$

If an impact is at a depth Z and the direction of a nonabsorbed neutron is $0 \le \theta < \pi/2$, $0 \le \phi < 2\pi$, then the probability that it will leave the slab (via $Z = 0$) with no more impacts is $e^{-\Sigma_t Z \sec\theta}$ and therefore, because of isotopy, $f(\phi, \theta)$ must be proportional to this. Restated with proportionality constant K,

$$f(\phi, \theta)\, d\Omega = K e^{-\Sigma_t Z \sec\theta}\, d\Omega$$

To evaluate K,

$$1 = \int_0^{\pi/2} \int_0^{2\pi} K e^{-\Sigma_t Z \sec\theta} \sin\theta \, d\phi \, d\theta$$

$$= 2\pi K \int_0^{\pi/2} e^{-\Sigma_t Z \sec\theta} \sin\theta \, d\theta$$

$$1 = 2\pi K \left[e^{-\Sigma_t Z} + \Sigma_t Z \, Ei(-\Sigma_t Z) \right]$$

$$K = \frac{1}{4\pi H(-\Sigma_t Z)}$$

where H is defined as

$$H(-\Sigma_t Z) = \frac{1}{2} \left[e^{-\Sigma_t Z} + \Sigma_t Z \, Ei(-\Sigma_t Z) \right]$$

The density function then becomes

$$f(\phi, \theta) = \frac{e^{-\Sigma_t Z \sec\theta}}{4\pi H(-\Sigma_t Z)}$$

This is the directional density of return for a neutron returning with its final impact having been at depth Z. The bottom of Table 4 estimates the breakdown of total albedo into the cell of final impact and thus gives depths Z roughly (within a half cell width). If it is now assumed that a neutron leaving a cell had its impact at the middle of the cell, then a rough estimate of α may be had by summing over all cells. Ideally, if $\phi(Z)\, dZ$ is the a priori probability that an incident neutron will eventually backscatter from a last impact at depth Z, then

$$\alpha(\theta, \phi) = \int_0^T \phi(Z)\, f(\phi, \theta)\, dZ$$

This will now be approximated roughly. The approximation may be improved ultimately by dividing the slab into more cells. The approximation will be made in Table 9 for various angles θ. To do this, $f(\phi, \theta)$ is computed and entered into the third column for various values of θ. The Q values of the fourth column have been taken from the last line of Table 4. The sum of the products, $Q_j f(\phi, \theta)$, then is an estimate of differential albedo. The results in Table 9 are plotted in Figure 2.

Using the results of Table 9, the density $g(\theta)$ is computed in Table 10 and plotted in Figure 3 as $\alpha\, g(\theta)$. The area under this curve is 0.1398, the estimated total albedo. It is noted that this function has a mode at about $\theta = 58$ degrees. This simply means that a greater percentage of the total albedo scatters back into a neighborhood $d\theta$ about an angle θ of approximately 58 degrees than into other neighborhoods of the same size. This does not say that differential albedo is maximum near 58 degrees; differential albedo has its maximum at $\theta = 0$ (Figure 2).

Computations for the scatter through angular distribution similar to those for albedo (Tables 9 and 10 and Figures 2 and 3) could be made using the same technique — a novel feature being that here $\pi/2 < \theta \leq \pi$. These computations were not actually performed for this example. The density function for total neutrons going through would contain the additional novel feature that there is a spike of mass (in this case 0.60653/0.73792) in the direction $-\theta_0 = \pi$, corresponding to the probability of passing directly through the slab with zero impacts.

There remains the task of estimating the expected number of neutron impacts and to analyze this into various classifications. There are two methods (say, A and B) available for estimating the expected number of impacts.

To explain method A, consider Table 3. For V_1 the sum of the components 1 through 5 (0.3934693) is the probability of experiencing a first impact. The probability of experiencing a second impact is the sum of the components 1 through 5 of V_2. Thus, if these component sums are summed over all V_i, $i = 1, 2, 3, \ldots$, this double summation will be an estimate of the expected number of total impacts.

Method B is a direct application of the definition of expected value, namely

$$\sum_{h=1}^{\infty} hP\,(\text{exactly } h \text{ impacts}) = E\,(\text{number of impacts})$$

To use this formula it will be necessary to estimate P (exactly h impacts) as the sum of the following three quantities: (1) probability of entering cell 0, having had exactly h impacts; (2) probability of entering cell 6, having had exactly h impacts; and (3) probability of being absorbed at the h^{th} impact.

By the nature of the convergences, method A should, in practice, be the more accurate, and method B will constitute an approximate check.

Table 9. Computation of Differential Albedo $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

θ	$\Sigma_t Z$	$f(\phi, \theta)$	Q	$Q f(\phi, \theta)$
0	0.05	0.1828782	0.04544	0.0083100
	0.15	0.2136937	0.03421	0.0073105
	0.25	0.2394104	0.02598	0.0062199
	0.35	0.2628340	0.01976	0.0051936
	0.45	0.2848776	0.01442	0.0041079
			$\alpha(\phi, 0)$ =	0.0311419
$\pi/9$	0.05	0.1822923	0.04544	0.0082834
	0.15	0.2116464	0.03421	0.0072404
	0.25	0.2355999	0.02598	0.0061209
	0.35	0.2569960	0.01976	0.0050782
	0.45	0.2767680	0.01442	0.0039910
			$\alpha(\phi, \pi/9)$ =	0.0307139
$2\pi/9$	0.05	0.1801068	0.04544	0.0081841
	0.15	0.2041250	0.03421	0.0069831
	0.25	0.2218114	0.02598	0.0057627
	0.35	0.2361884	0.01976	0.0046671
	0.45	0.2482972	0.01442	0.0035804
			$\alpha(\phi, \pi/9)$ =	0.0291774
$5\pi/18$	0.05	0.1778666	0.04544	0.0080823
	0.15	0.1966027	0.03421	0.0067258
	0.25	0.2083561	0.02598	0.0054131
	0.35	0.2163763	0.01976	0.0042756
	0.45	0.2218461	0.01442	0.0031990
			$\alpha(\phi, 5\pi/18)$ =	0.0276958

Continued

Table 9. Continued.

θ	$\Sigma_t Z$	$f(\phi, \theta)$	Q	$Q\,f(\phi, \theta)$
$\pi/3$	0.05	0.1739591	0.04544	0.0079047
	0.15	0.1839279	0.03421	0.0062922
	0.25	0.1864530	0.02598	0.0048440
	0.35	0.1852160	0.01976	0.0036599
	0.45	0.1816460	0.01442	0.0026193
			$\alpha(\phi, \pi/3)$ =	0.0253201
$7\pi/18$	0.05	0.1661066	0.04544	0.0075479
	0.15	0.1601279	0.03421	0.0054780
	0.25	0.1480024	0.02598	0.0038451
	0.35	0.1340471	0.01976	0.0026488
	0.45	0.1198628	0.01442	0.0017284
			$\alpha(\phi, 7\pi/18)$ =	0.0212482
$4\pi/9$	0.05	0.1441539	0.04544	0.0065504
	0.15	0.1046612	0.03421	0.0035805
	0.25	0.0728561	0.02598	0.0018928
	0.35	0.0496974	0.01976	0.0009820
	0.45	0.0334688	0.01442	0.0004826
			$\alpha(\phi, 4\pi/9)$ =	0.0134883
$17\pi/36$	0.05	0.1083248	0.04544	0.0049223
	0.15	0.0444110	0.03421	0.0015193
	0.25	0.0174572	0.02598	0.0004535
	0.35	0.0067243	0.01976	0.0001329
	0.45	0.0025571	0.01442	0.000369
			$\alpha(\phi, 17\pi/36)$ =	0.0070649

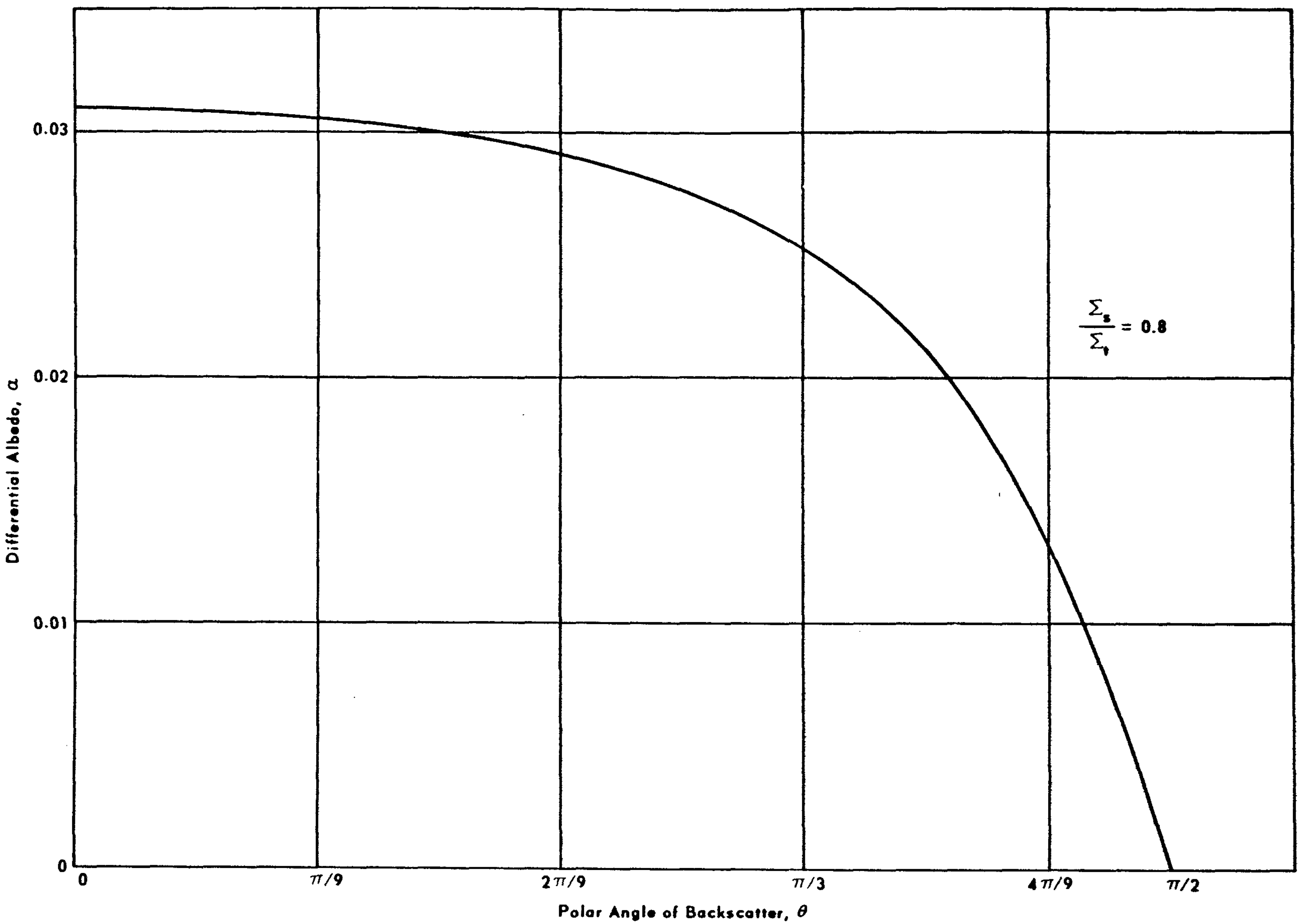

Figure 2. Differential albedo versus θ.

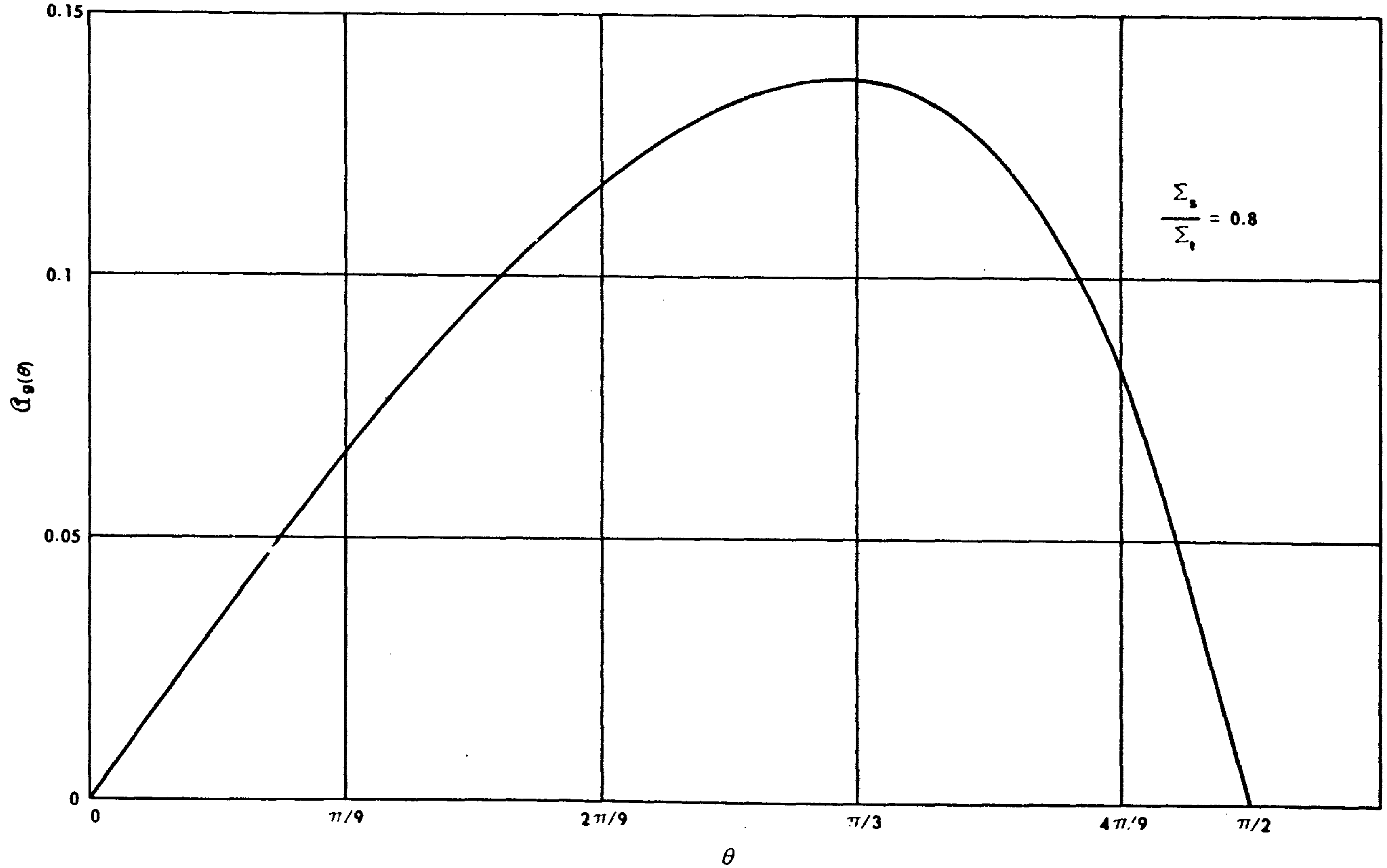

Figure 3. $\mathcal{Q}\,g(\theta)$ versus θ.

Table 10. The Density $g(\theta)$, $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

θ	$2\pi \sin\theta$	α	$\mathcal{A}\, g(\theta)$	$g(\theta)$
0.0	0.0	0.0311419	0.0	0.0
$\pi/9$	2.148976	0.0307139	0.06600	0.47214
$2\pi/9$	4.038754	0.0291774	0.11784	0.84298
$5\pi/18$	4.813199	0.0276958	0.13331	0.95364
$\pi/3$	5.441398	0.0253201	0.13778	0.98562
$7\pi/18$	5.904263	0.0212482	0.12545	0.89742
$4\pi/9$	6.187730	0.0134883	0.08346	0.59704
$17\pi/36$	6.259276	0.0070649	0.04422	0.31633
$\pi/2$	6.283185	0.0	0.0	0.0

Proceeding with method A, Table 11 is prepared from Table 3 by computing the sums

$$\sum_{j=1}^{5} V_{ij}$$

and then finally estimating the sum of all these, $i = 1, 2, \ldots,$ obtaining the result 0.612 for the expected total number of impacts in the offing for a neutron directed at the slab.

Proceeding with method B, Table 12 is constructed by transcribing Tables 6, 7, and 8 to columns 2, 3 and 4 of Table 12, summing these to form the column "Probability of exactly h impacts," and then forming the last column by the indicated multiplication.

The reason the result of Table 12 is intrinsicly less accurate than that of Table 11 is that the figure 0.0150 in the lower right corner of Table 12 is a rough infinite-sum estimate, made rough because the ratios of successive entries in the last column stabilize much less rapidly than those of Table 11.

A breakdown of the expected 0.61233 impacts can be approximated as shown in Table 13. The expected number of absorbing impacts is computed simply as $0.2 \times 0.61233 = 0.12247$. To compute the expected number of impacts invested in absorbed neutrons, the cross-products of the first and third column of Table 8 are summed. Here again the slow convergence introduces inaccuracy in the estimate

for $i = 6, 7, \ldots$. Similarly, Tables 6 and 7 are used to estimate the expected number of impacts of the incident neutron to be invested, respectively, in albedo and scatter-through. The Table 13 round-off error, 0.61310 versus 0.61233, could be reduced by extending Table 3 with some more matrix multiplications, computing, say, V_6, V_7, and V_8.

If a neutron is backscattered, the expected number of impacts it has is

$$E(N_b) = \frac{0.21612}{0.13979} = 1.546$$

If a neutron has been absorbed, the expected number of impacts it has had is

$$E(N_o) = \frac{0.19057}{0.12229} = 1.558$$

If a neutron is scattered through, the expected number of impacts it has had is

$$E(N_s) = \frac{0.20641}{0.13139} = 1.571$$

Table 11. Expected Number of Impacts $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)^*$

i	P(i^{th} impact)
1	0.3934693
2	0.1403356
3	0.0503283
4	0.0180714
5	0.0064908
6	0.002331 (est.)
7	0.000837 (est.)
8, 9, ...	0.00047 (est.)
$\sum_{i=j}^{\infty} i$	0.61233 (est.)

* Probability of experiencing the i^{th} impact is computed as $\sum_{j=1}^{5} v_{ij}$ (from Table 3).

Estimated expected number of impacts is 0.61233.

Table 12. Check Estimate of Expected Number of Impacts $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

h	(1) With Exactly h Impacts			(2) Probability of Exactly h Impacts	(1) · (2)
	P(Backscatter)	P(Scatter-Through)	P(A' orbed)		
1	0.0908481	0.0835917	0.0786939	0.2531337	0.2531337
2	0.0314616	0.0304791	0.0280666	0.0900073	0.1800146
3	0.0111631	0.0110282	0.0100657	0.0322569	0.0967707
4	0.0039925	0.0039739	0.0036143	0.0115807	0.0463227
5	0.0014319	0.0014293	0.0012981	0.0041593	0.0207965
6, 7, ... Est	0.00089	0.00089	0.00055	0.00233	0.0150
Expected total number of impacts					0.6120 Est

Table 13. Analysis of Expected Total Number of Impacts $\left(\frac{\Sigma_s}{\Sigma_t} = 0.8\right)$

Expected number of absorbing impacts	0.12247	
Expected number of impacts of neutron before absorbing impact	0.06810	
Expected number of impacts invested in absorption		0.19057
Expected number of impacts invested in albedo		0.21612
Expected number of impacts invested in scatter-through		0.20641
Total expected impacts		0.61310

THE PERFECT SCATTERING MEDIUM

For the numerical example solved in the preceding section, it will be recalled that $\Sigma_s/\Sigma_t = 0.8$; i.e., the slab material could absorb neutrons, as well as scatter them. The next case to be considered is the case of the perfect scatterer. That is,

$$\Sigma_s = \Sigma_t$$

$$\Sigma_a = 0$$

As before, we choose

$$T = \frac{1}{2\Sigma_t}$$

$$h = \frac{1}{10\Sigma_t}$$

$$\theta_o = 0$$

$$j = 0, 1, 2, 3, 4, 5, 6$$

Since the computations are similar, only results will be presented. Tables 2A through 7A, corresponding to Tables 2 through 7, give results for the perfect scattering case. Figure 2A displays differential albedo for the perfect scattering case.

The expected number of collisions of a neutron before it leaves the slab is found to be

$$E(N) = 0.711$$

The expected number of collisions of a neutron which has backscattered from the slab is

$$E(N_b) = 1.792$$

The expected number of collisions of a neutron which is scattered through the slab, i.e., has suffered at least one impact while penetrating, is

$$E(N_s) = 1.824$$

The expected number of collisions of a neutron which penetrates the slab, with or without collisions, is

$$E(N_t) = 0.437$$

Table 2A. Transition Matrix $\left(\Sigma_s = \Sigma_t\right)$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4139172 & 0.1721655 & 0.0933979 & 0.0616543 & 0.0455087 & 0.0352418 & 0.1781145 \\
0.3205194 & 0.0933979 & 0.1721655 & 0.0933979 & 0.0616543 & 0.0455087 & 0.2133564 \\
0.2588651 & 0.0616543 & 0.0933979 & 0.1721655 & 0.0933979 & 0.0616543 & 0.2588651 \\
0.2133564 & 0.0455087 & 0.0616543 & 0.0933979 & 0.1721655 & 0.0933979 & 0.3205194 \\
0.1781145 & 0.0352418 & 0.0455087 & 0.0616543 & 0.0933979 & 0.1721655 & 0.4139172 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Table 3A. Successive State Vectors ($\Sigma_s = \Sigma_t$)

V_1	0	0.0951626	0.0861066	0.0779125	0.0704982	0.0637894	0.6065307
V_2	0.1135602	0.0346859	0.0382389	0.0378405	0.0350116	0.0296427	0.7110203
V_3	0.1627189	0.0145142	0.0168657	0.0173224	0.0162667	0.0136691	0.7586440
V_4	0.1845217	0.0063641	0.0075021	0.0078144	0.0073955	0.0062196	0.7801833
V_5	0.1942691	0.0028339	0.0033548	0.0035126	0.0033362	0.0028090	0.7898852
V_6	0.1986388						0.7942470
⋮							
V_∞ (est.)	0.2021809	0	0	0	0	0	0.7978191

Table 4A. Probability of Neutron's Entering Zeroth Cell Having Had i Impacts With i^{th} Impact Occurring in Cell j ($\Sigma_s = \Sigma_t$)

i	j					Total
	1	2	3	4	5	
1	0.0393894	0.0275988	0.0201688	0.0150412	0.0113618	
2	0.0143571	0.0122563	0.0097956	0.0074699	0.0052798	
3	0.0060077	0.0054058	0.0044842	0.0034706	0.0024347	
4	0.0026342	0.0024046	0.0020229	0.0015779	0.0011078	
5	0.0011730	0.0010753	0.0009093	0.0007118	0.0005003	
	$\sum_{i=1}^{5} p_{ij}$					
	0.0635614	0.0487408	0.0373808	0.0282714	0.0206844	0.1986388
	Est $\sum_{i=1}^{\infty} p_{ij} = Q_j$					
	0.0645274	0.0496263	0.0381053	0.0288349	0.0210869	0.2021809

Table 5A. Probability of Neutron's Entering Sixth Cell, Having Had i Impacts With i^{th} Impact Occurring in Cell j ($\Sigma_s = \Sigma_t$)

i	j = 0	1	2	3	4	5	Total
0	0.6065307	0.0	0.0	0.0	0.0	0.0	
1	0.0	0.0169498	0.0183714	0.0201688	0.0225960	0.0264035	
2	0.0	0.0061780	0.0081585	0.0097956	0.0112219	0.0122696	
3	0.0	0.0025852	0.0035984	0.0044842	0.0052138	0.0056579	
4	0.0	0.0011335	0.0016006	0.0020229	0.0023704	0.0025744	
5	0.0	0.0005048	0.0007159	0.0009093	0.0010693	0.0011627	
$\sum_{i=1}^{5} p_{ij}$							
	0.6065307	0.0273514	0.0324447	0.0373808	0.0424714	0.0480681	0.7942470
Est $\sum_{i=6}^{\infty} p_{ij}$							
		0.0005205	0.0006174	0.0007113	0.0008082	0.0009147	0.0035721
Est $\sum_{i=1}^{\infty} p_{ij}$							
	0.6065307	0.0278719	0.0330621	0.0380921	0.0432796	0.0489828	0.7978191

Table 6A. Number of Impacts Analysis of Albedo (Zeroth Cell Entrants) ($\Sigma_s = \Sigma_t$)

Number of Impacts	Probability
1	0.1135602
2	0.0491587
3	0.0218029
4	0.0097473
5	0.0043697
Total of 5	0.1986388
6 and more (est.)	0.003542
Estimated total albedo	0.202181

Table 7A. Number of Impacts Analysis of Neutrons Transmitted Through Slab ($\Sigma_s = \Sigma_t$)

Number of Impacts	Probability
0	0.6065307
1	0.1044896
2	0.0476236
3	0.0215394
4	0.0097019
5	0.0043618
Total of 5	0.7942470
6 and more (est.)	0.003572
Estimated total transmission	0.797819

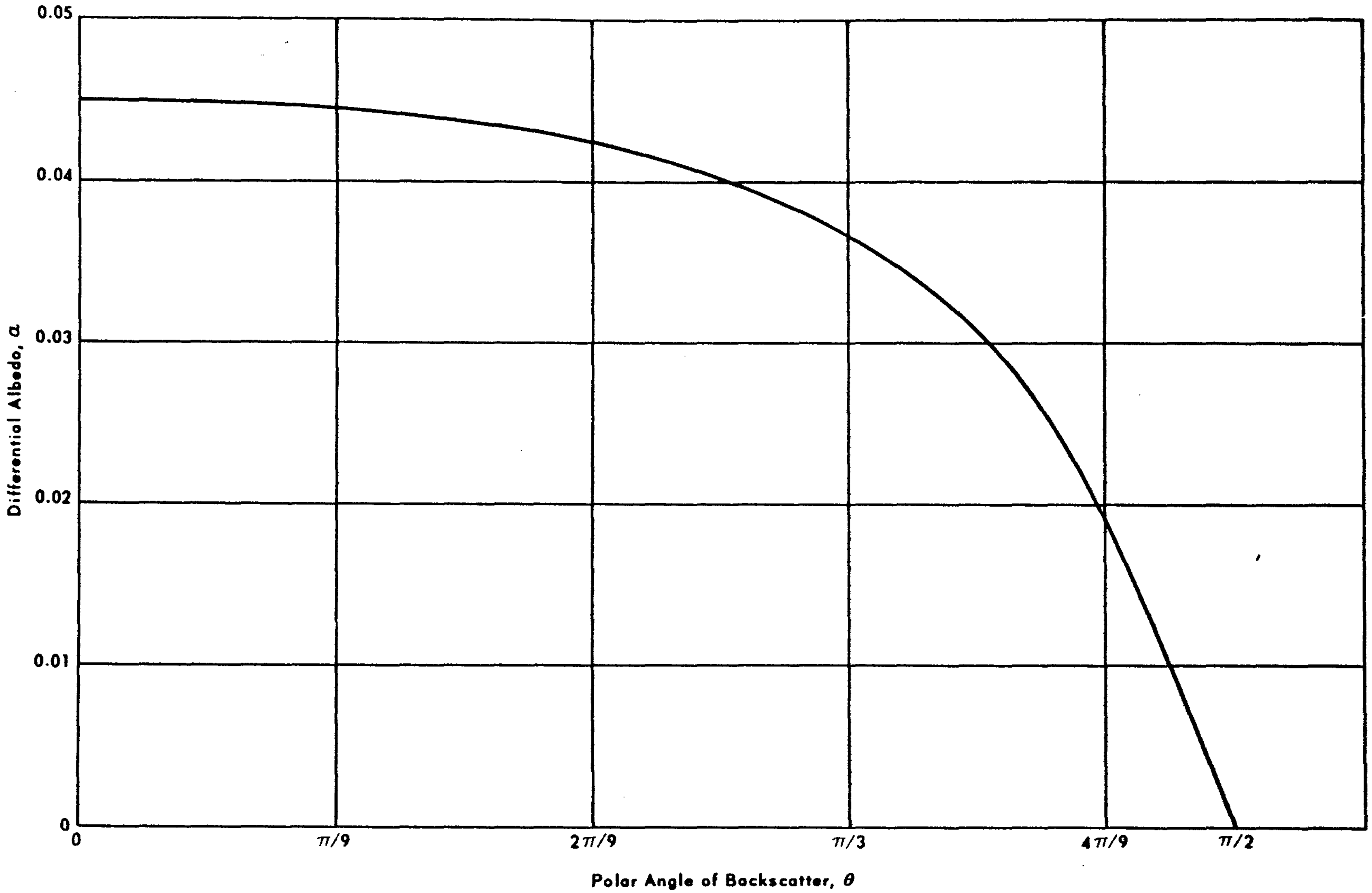

Figure 2A. Differential albedo for perfect scattering medium.

SUMMARY

The two cases that have been considered have both involved a neutron normally incident on a plane slab with a thickness of one-half mean free path. Using a discrete-ordinate approximation, the neutron-diffusion problem has been solved by employing a Markov chain process to represent the random walk phenomenon of isotropic scattering, with or without absorption.

The 15 questions posed in the section Statement of the Problem have been solved, as reviewed below:

1. For $\Sigma_s/\Sigma_t = 0.8$, the total albedo $\mathcal{A}$ was estimated to be 0.13979. For $\Sigma_s = \Sigma_t$, $\mathcal{A} = 0.20218$.

2. The estimated differential albedo α for the two cases is shown in Figures 2 and 2A.

3. The analyses of total albedo into number of impacts are presented in Tables 6 and 6A.

4. The analyses of total albedo into depth probability density of last impact before backscatter from the slab are given in Tables 4 and 4A.

5. The probability of absorption for the case $\Sigma_s/\Sigma_t = 0.8$ was estimated to be $\mathcal{A}^\circ = 0.12229$. For the perfect scattering case, clearly the probability of absorption is zero.

6. The analysis of $\mathcal{A}^\circ$ into number of impacts for the case where $\Sigma_s/\Sigma_t = 0.8$ is shown in Table 8.

7. The analysis of $\mathcal{A}^\circ$ into depth density of absorption point for the case where $\Sigma_s/\Sigma_t = 0.8$ is also shown in Table 8.

8. The probabilities of transmission, with or without collision, were estimated to be 0.7392 for $\Sigma_s/\Sigma_t = 0.8$ and 0.79782 for $\Sigma_s = \Sigma_t$.

9. The analyses of transmitted neutrons into number of collisions are given in Tables 7 and 7A.

10. The analyses of neutrons scattered through into depth density of last impact before penetration are given in Tables 5 and 5A.

11 - 15. The expected numbers of impacts were estimated to be

	$\Sigma_s/\Sigma_t = 0.8$	$\Sigma_s = \Sigma_t$
$E(N)$	0.612	0.711
$E(N_b)$	1.546	1.792
$E(N_s)$	1.571	1.824
$E(N_t)$	0.280	0.437
$E(N_o)$	1.558	—

where the subscripts b, s, t, and o refer to backscatter, scatter through, transmission, and absorption, respectively.

All of the numerical values obtained are certain to be somewhat in error because of the approximation of assuming that all collisions occur at midpoints of the slab layers. The approximation can be improved by increasing the number of subdivisions or layers. A consequence will be that the transition matrix will need to become larger.

PLANS FOR THE FUTURE

It is planned to devise and employ a computer program to perform calculations with increasingly fine slab layers. Results will be examined for convergence. If results for infinitely thin layers can be approximated by extrapolation, the problem will be solved to a high order of approximation.

The computer program will also be used to generate albedo data for slabs of various thicknesses, for different values of Σ_s/Σ_t, and for various polar angles of neutron incidence.

Finally, attempts will be made to treat more difficult problems, such as the case where neutron scattering is not isotropic, and, if possible, the case where Σ_s and Σ_t are functions of the neutron energy, which changes with each scattering event.

Albedo values calculated by the Markov matrix method will be compared with available experimental values. If agreement is satisfactory for known cases, albedo values can be generated for use in shielding problems where backscatter is important. Also, the method can readily be generalized to treat cases of penetration through thin slabs of scattering and absorbing shields.

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GLOSSARY OF SYMBOLS

Total albedo; i.e., probability that eventually the neutron backscatters out of the slab. This is, of course, a function of Σ_a, Σ_s, T, and θ_0.

Total absorption; i.e., probability that eventually the neutron is absorbed. This is, of course, a function of Σ_a, Σ_s, T, and θ_0.

Total transmission; i.e., probability that eventually the neutron is transmitted through the slab. This is, of course, a function of Σ_a, Σ_s, T, and θ_0.

Probability that the neutron passes through the slab with no impacts; i.e., uncollided transmission. $\mathcal{A}_0^t = e^{-T\Sigma_t \sec\theta_0}$.

Back hemisphere in region Z < 0, with base in plane Z = 0.

Differential solid angle. $d\Omega = \sin\theta \, d\theta d\phi$.

Expected value operator. Thus, E(N) simply denotes the expected (or average) number of impacts to be experienced by the neutron, where average is taken over an infinite number of trials of the experiment: "shoot the neutron at the slab under the given conditions."

Directional (ϕ, θ), two-dimensional, density of the backscatter from a fixed known depth, Z, where $\int_{BHS} f(\phi, \theta) d\Omega = 1$.

Directional (θ), one-dimensional, density of backscatter, where $\int_0^{\pi/2} g(\theta) d\theta = 1$.

Sub-slab thickness; so that T/h is a positive integer number of sub-slabs, all of equal thickness; and the number of Markov states is (2 + T/h). The symbol h is also used as an index for counting impacts.

Probability density function of W projection of next impact.

Probability distribution function, where dH(W)/dW = h(W).

Markov state indices, where i, j = 0, 1, 2, . . . , T/h, T/h + 1. Also, i, j are used to count impacts.

A proportionality constant used to compute $f(\phi, \theta)$.

Entry in M at i^{th} row and j^{th} column.

Transition matrix.

Total number of neutron impacts. Thus, $N \geq 0$.

If eventually the neutron backscatters to Z < 0, this represents the number of impacts it has experienced. Thus, $N_b \geq 1$.

N_a — If the neutron is absorbed, this represents the number of impacts it has experienced. Thus, $N_a \geq 1$.

N_s — If eventually the neutron scatters through the slab, this represents the number of impacts it has experienced. Thus, $N_s \geq 1$.

N_t — If eventually the neutron goes through the slab, this represents the number of impacts it has experienced. Thus, $N_t \geq 0$.

P_{ij} — Probability of entering zeroth (sixth) cell, having had i impacts with the i^{th} impact occurring in cell j.

P(·) — Probability of (·). For example, P(j) denotes probability that the neutron will experience exactly j impacts.

Q_j — Probability of backscattering with the last impact in the j^{th} cell.

S — Distance (scaler) measured in same units as the Cartesian coordinates.

T — Known slab thickness. Thus, space occupied by the slab is $(-\infty < X < \infty)$, $(-\infty < Y < \infty)$, $(0 \leq Z \leq T)$.

v_{ij} — j^{th} cell entry of i^{th} state vector.

V — $\Sigma_t W \sec\beta$.

V_i — i^{th} state vector.

W — An arbitrarily directed one-dimensional coordinate system in the same distance units as the Cartesian system, and with origin at an impact point of the neutron.

X, Y, Z — A fixed set of right-handed Cartesian axes with the initial slab face being the (X, Y) plane and the -Z axis being totally outside the slab.

α — Differential albedo per steradian; a function of Σ_a, Σ_s, T, θ_0, and θ, where the integral over the backward (Z < 0) hemisphere $\int_{BHS} \alpha d\Omega = \mathcal{A}$ or, equivalently, $\int_0^{2\pi} \int_0^{\pi/2} \alpha \sin\theta \, d\theta \, d\phi = \mathcal{A}$.

α^t — Differential scatter-through (collided transmission) per steradian; a function of Σ_a, Σ_s, T, θ_0, and θ, where the integral over the forward (Z > T) hemisphere $\int_{FHS} \alpha^t d\Omega = \mathcal{A}^t - \mathcal{A}_0^t$ or, equivalently, $\int_0^{2\pi} \int_{\pi/2}^{\pi} \alpha^t \sin\theta \, d\theta d\phi = \mathcal{A}^t - \mathcal{A}_0^t$.

β — Angle between a ray and the positive W axis.

η — Angle between projection of ray onto a plane perpendicular to the W axis at W = 0 and a fixed direction in this plane.

GLOSSARY OF SYMBOLS

If the neutron is absorbed, this represents the number of impacts it has experienced. Thus, $N_o \geq 1$.

If eventually the neutron scatters through the slab, this represents the number of impacts it has experienced. Thus, $N_s \geq 1$.

If eventually the neutron goes through the slab, this represents the number of impacts it has experienced. Thus, $N_t \geq 0$.

Probability of entering zeroth (sixth) cell, having had impacts with the ith impact occurring in cell j.

Probability of (·). For example, P(j) denotes probability that the neutron will experience exactly j impacts.

Probability of backscattering with the last impact n the jth cell.

Distance (scaler) measured in same units as the Cartesian coordinates.

Known slab thickness. Thus, space occupied by the slab is $(-\infty < X < \infty)$, $(-\infty < Y < \infty)$, $(0 \leq Z \leq T)$.

th cell entry of ith state vector.

$\Sigma_t W \sec \beta$.

th state vector.

An arbitrarily directed one-dimensional coordinate system in the same distance units as the Cartesian system, and with origin at an impact point of the neutron.

A fixed set of right-handed Cartesian axes with the initial slab face being the (X, Y) plane and the -Z axis being totally outside the slab.

Differential albedo per steradian; a function of Σ_a, Σ_s, T, θ_o, and θ, where the integral over the backward (Z < 0) hemisphere $\int_{BHS} \alpha d\Omega = \mathcal{A}$ or, equivalently, $\int_0^{2\pi} \int_0^{\pi/2} \alpha \sin\theta \, d\theta \, d\phi = \mathcal{A}$.

Differential scatter-through (collided transmission) per steradian; a function of Σ_a, Σ_s, T, θ_o, and θ, where the integral over the forward (Z > T) hemisphere $\int_{FHS} \alpha^t d\Omega = \mathcal{A}^t - \mathcal{A}^t_o$ or, equivalently, $\int_0^{2\pi} \int_{\pi/2}^{\pi} \alpha^t \sin\theta d\theta d\phi = \mathcal{A}^t - \mathcal{A}^t_o$.

Angle between a ray and the positive W axis.

Angle between projection of ray onto a plane perpendicular to the W axis at W = 0 and a fixed direction in this plane.

θ, ϕ Polar and azimuthal angles of neutron return — if it does return. If X', Y', Z' is a new set of axes (old set translated but not rotated), having its origin at the neutron return point, then θ is the polar angle that the returned neutron makes with -Z' axis, and ϕ is the azimuthal angle that the returned neutron makes with -X' axis; where, respectively, the directions -X', Y', X', and -Y' have azimuths ϕ equal to 0, $\pi/2$, π, and $3\pi/2$. When $\theta = 0$, ϕ is undefined.

θ_o Polar angle of neutron incidence. Neutron is directed from outside the slab without interference toward the Cartesian origin (0, 0, 0). This direction of flight is such that it makes angle θ_o with -Z axis, angle $(\pi/2 - \theta_o)$ with -X axis, and angle $\pi/2$ with Y axis; $0 \leq \theta_o \leq \pi/2$.

$\Sigma_t, \Sigma_s, \Sigma_a$ Known constants; respectively, total, scattering, and absorption macroscopic cross sections of slab material. $\Sigma_t = \Sigma_s + \Sigma_a$. Distance traveled by the neutron within the slab between two successive impacts is a random variable with statistical density function, $f(S) = \Sigma_t e^{-\Sigma_t S}$. A priori probability of being absorbed at the next impact is Σ_a / Σ_t. The units for each Σ are the same as for 1/S.

ϕ_o Azimuthal angle of neutron incidence; by definition, always zero except when $\theta_o = 0$, and then ϕ_o is undefined.

Ω Solid angle measured in steradians; where, e.g., $\Omega = 2\pi$ for a hemisphere.

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The concept of a Markov chain has been used to treat the random processes of scatter and absorption which occur when neutrons are incident on a slab of finite thickness, such as a shield or an inside wall of a shelter entranceway. It is assumed that scattering is isotropic in the laboratory system and that the scattering and absorption cross sections do not change during a neutron-scattering history. The encouraging results obtained to date indicate that for thin slabs the analytical random walk method may have important advantages over Monte Carlo calculations (which require lengthy computer runs to obtain acceptably small statistical variances), moments method calculations (which are actually appropriate only for infinite medium cases), and numerical solutions of the neutron transport equation (which are lengthy, tedious, and necessarily approximate).

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